**Banknotes, liquidity insurance, and the role of interbank settlement**

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**Abstract**

This paper considered how an interbank settlement system could improve social welfare. When only banknotes are able to circulate, the consumption by agents may fluctuate so that welfare decreases. However, when both banknotes and fiat money are able to circulate, agents can avoid these fluctuations in consumption, but only when the central bank acts as the LLR in which case the central bank should maintain the cost needed to sustain the interbank settlement system. Finally, when liquidity insurance is introduced, not only the consumption level of agents is smoothed, but the central bank can avoid the costs of sustaining the interbank settlement system.

**Key words:** interbank settlement system, payment economics

## Introduction

This paper considers how the interbank settlement system can improve transactions between agents and whether it improves social welfare from the viewpoint of payment economics. Kahn and Roberds (2008, p2) state that ``Payments and payment systems, so ubiquitous and obviously essential to real-world economies, are conspicuously absent from the world of Arrow–Debreu." They also declare that the fundamental imperfections in payments are twofold. The first is the time mismatch in consumption. The second is the imperfect enforcement of credit arrangements.

Recently, Cavalcanti and Wallace (1999), Williamson (1999), Kiyotaki and Moore (2000), and Andolfatto and Nosal (2003) consider the situation where banknotes can circulate. In these models, some agents with special skills, such as in obtaining public trading histories (Cavalcanti and Wallace, 1999), act as a bank and can issue banknotes. Alternatively, Freeman (1996, 1999) and Zhou (2000) consider the role of the monetary authority as a liquidity provider when banknotes and fiat money coexist in the same economy. Nevertheless, most of these studies consider only the behavior of a single bank. Therefore, they do not consider how the interbank settlement system can affect bank liquidity shortages or how monetary policy transmits through the interbank settlement system. By extending the model in Kahn and Roberds (1999) in this paper, we consider how the interbank settlement system can affect transactions between agents and how the interbank settlement system potentially affects social welfare.

In our model, there are two types of agents. One type holds banknotes and wants to consume the bank's product. The other type of agent holds banknotes but does not want to consume the bank's product but instead wants to consume another bank's product. In this case, the agents should exchange banknotes with those agents that hold the banknotes of the bank whose product they wish to consume. The population share that prefers the production of each of these banks then determines the banknote exchange rate.

In this situation, we derive that when only banknotes can circulate, the consumption level of agents may fluctuate because the banknote exchange rate between banks also fluctuates. However, when we introduce a central bank such that the interbank settlement system can operate, agents can avoid any fluctuations in consumption because product purchase is through the interbank settlement system, such that the banknote exchange rate does not fluctuate. However, to sustain this system, the central bank may have to act as the lender of last resort (LLR). We derive that when the central bank acts as the LLR, agents can avoid fluctuations in their level of consumption, although the central bank should bear the costs. Finally, with the introduction of a system of liquidity insurance, any bank liquidity shortage is fully covered such that not only agents can avoid fluctuations in their levels of consumption, but the central bank can avoid the costs needed to sustain the interbank settlement system.

The remainder of the paper is structured as follows. In Section 2, we describe the model. In Section 3, we derive the level of social welfare when only banknotes are able to circulate in the economy. In Section 4, we derive the level of social welfare when both banknotes and fiat money are able to circulate and payments may settle through the interbank settlement system. In Section 5, we introduce liquidity insurance and show that this economy can achieve the highest level of social welfare.

## The model

In our model, there are two regions, A and B.[1] In each region, there are three types of agents: consumers, merchants, and a bank. Each agent lives for three periods and has a risk-averse utility function. We normalize the population of consumers and merchants in each region to unity. At $t$ = 0, the consumers are endowed with one unit of goods ($c_i$) ($i$ = A, B) and the merchants are endowed with one unit of goods ($m_i$). The consumer in region A (B) then wishes to consume $m_A$ ($m_B$) at $t$ = 1. However, the merchant in region A does not value the consumer's good. Therefore, there is a lack of the double coincidence of wants. However, a

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1 This model is very similar in spirit to Kahn and Roberds (1999). However, to best consider the role of the interbank settlement system, we consider the situation where the two banks are located in different regions.

bank can produce an output valued by merchants in that a bank in region A (B) can produce an output $b_A$ $(b_B)$ at $t$ = 2 if a consumer's endowment $c_A$ $(c_B)$ is invested at $t$ = 0. The production technology of the bank in A (B) allows it to produce one unit of $b_A$ $(b_B)$ from one unit of the respective consumer's endowment good $c_A$ $(c_B)$. At $t$ = 1, the merchant receive a shock to its preference at the beginning of $t$ = 1. That is, a fraction $(1-\alpha)$ of merchants in region A know that they will want to consume the output of bank A $(b_A)$ at $t$ = 2 and a fraction $\alpha$ of merchants know that they will want to consume the output of bank B $(b_B)$ at $t$ = 2. In addition, a fraction $(1-\beta)$ of merchants in region B know that they will want to consume the output of bank B $(b_B)$ at $t$ = 2 and a fraction $\beta$ know that they will want to consume the output of bank A $(b_A)$ at $t$ = 2. We assume $\alpha$ and $\beta$ are identically and independently distributed and each agent knows the prior distribution of $\alpha$ and $\beta$. To simplify the following discussion, we assume that $\alpha$ $(\beta)$ takes a value of 1/3 (2/3) with a probability of 1/2.

**Payment by banknotes**

In this section, we consider the situation where the payment for all transactions is with banknotes.[2] At $t$ = 0, a consumer in region A invests endowment $c_A$ in return for banknote $D_A$ of bank A. Bank A can issue its banknote $D_A$ because it can produce $b_A$ at $t$ = 2. In other words, the holder of one unit of $D_A$ can exchange one unit of $b_A$ for it at $t$ = 2. At $t$ = 1, the consumer in region A obtains the merchant's endowment $m_A$ by paying for it with banknote $D_A$. At $t$ = 2, a merchant in region A who wishes to consume the output produced by bank A (for subsequent explanation, we denote this merchant $M_A^a$) receives $b_A$ by paying $D_A$ to bank A.

Alternatively, a merchant in region A who wishes to consume the output produced by bank B (denoted $M_A^b$) should meet with a merchant in region B who wishes to consume the output produced by bank B, $b_B$, (denoted $M_B^a$) in order to exchange banknote $D_A$ with banknote $D_B$. In region B, the same transactions also take place. Figure 1 shows all transactions in region A and B and indicates that all transactions are settled with banknotes. To consider the efficiency of this economy, we categorize the situation into four cases. Note that in every case, all consumers can receive the merchant's good. For this reason, to consider the efficiency of this economy, we focus on the consumption level of merchants.

(1) Case 1: $\alpha = \beta = 1/3$

In this case, in order to obtain $b_B$ at $t$ = 2, merchants in region A who want to consume the good produced by bank B (i.e., $M_A^b$ ) exchange $D_A$ with $D_B$ with a merchant in region B who wants to consume the output produced by bank A (i.e., $M_B^a$). As $M_B^a$ also wants $D_A$ in order to

[2] The transactions in region B are then mirror images of those in region A.

obtain $b_A$ at $t$ = 2, there exists a double coincidence of wants between $M_A^b$ and $M_B^a$. Further, given $\alpha = \beta$, the number of $M_A^b$ is the same as that of $M_B^a$. Therefore, $D_A = D_B$ holds. That is, all merchants in region A and B can obtain all the goods they want at $t$ = 2. Consequently, in this situation the utility of all merchants is $\mathrm{u}(b_A) = \mathrm{u}(b_B) = \mathrm{u(b)}$.[3] Given $\alpha = \beta = 1/3$, all demands for $\mathrm{b_A}$ and $\mathrm{b_B}$ equal one and the markets for $b_A$ and $b_B$ clear.

(2) Case 2: $\alpha = \beta = 2/3$

As in the previous case, the number of $M_A^b$ is again the same as the number of $M_B^a$. Therefore, the exchange rate of $D_A$ for $D_B$ is $D_A = D_B$. Therefore, the utility of all merchants is $\mathrm{u}(b_A) = \mathrm{u}(b_B) = \mathrm{u(b)}$.

(3) Case 3: $\alpha = 2/3,\ \beta = 1/3$

In this case, $M_A^b$ is double that of $M_B^a$. Therefore, the exchange rate of $D_A$ for $D_B$ is $D_B = 2D_A$. Thus, $M_B^a$ can receive $2D_A$ in return for giving $D_B$ to $M_A^b$ so that they can obtain two units of $b_A$ at $t$ = 2. Therefore, their utility level is $\mathrm{u}(2b_A) \equiv u(2b)$. However, given $D_A = D_B/2, M_A^b$ can obtain only half a unit of $b_B$ such that its utility is $\mathrm{u}(b_B/2) \equiv \mathrm{u(b/2)}$. Note that the demand for $b_A$ is the sum of the demands by $M_A^a$ who require one unit of $b_A$ and the demands by $M_B^a$ who require two units of $b_A$. As the number of $M_A^a$ is $1 - \alpha$ and the number of $M_B^a$ is $\beta$, the total demand for $b_A$ is $1 - \alpha + 2\beta$. In this case, $\alpha = 2/3$ and $\beta = 1/3$, therefore the demand for $b_A$ is one. All demands are satisfied. In the same way, the demand for $b_B$ is $1 - \beta + \frac{\alpha}{2} = 1$ so that the demand for $b_B$ also clears.

(4) Case 4: $\alpha = 1/3,\ \beta = 2/3$

This is the opposite to the previous case. Accordingly, $M_A^b$ receives $2b_B$ and its utility is $\mathrm{u}(2b_B)$. And $M_B^a$ receives $b_A/2$ and its utility is $\mathrm{u}(b_B/2)$. In this case, as in the previous case, the markets for both $b_A$ and $b_B$ clear.

As a result, the expected utility of the merchant in region A at $t$ = 0 $E_A[u_A]$ is

$$E_A[u_A] = \frac{1}{4}\,u(b) + \frac{1}{4}\,u(b) + \frac{1}{4}\left\{\frac{1}{3}u(b) + \frac{2}{3}u\left(\frac{b}{2}\right)\right\} + \frac{1}{4}\left\{\frac{2}{3}u(b) + \frac{1}{3}u(2b)\right\}$$

$$= \frac{3}{4}u(b) + \frac{1}{6}u\left(\frac{b}{2}\right) + \frac{1}{12}u(2b) \qquad (1)$$

The first and second terms on the right-hand side of (1) denote the expected utility level for

[3] Suppose that if the merchant receives a good which the merchant does not value (i.e., $M_A^b$ receives $b_A$ or $M_B^a$ receives $b_B$), the utility level of the merchant is then zero. We also assume that the utility function is identical across all merchants.

cases 1 and 2. The third term denotes the expected utility level of case 3 in which a merchant in region A wants to consume the good of bank A and B with a probability of 1/3 and 2/3, respectively. From this equation, it is obvious that in the situation where only banknotes circulate, the consumption level of the merchant fluctuates.[4] This is because, from cases 3 and 4, the exchange rate of $D_A$ for $D_B$ depends on the realization of $\alpha$ and $\beta$. Given the merchant is risk averse, the fluctuation in consumption decreases the merchant's expected utility.

**Fiat money with banknotes**

Next, consider the situation where not only banknotes but also fiat money can circulate in the economy. To consider this situation, we introduce a central bank into the model. The role of the central bank is to provide fiat money to the banks at $t = 0$. The central bank can do this because the central bank can evaluate the products of both banks. In other words, if both banks cannot repay the fiat money to the central bank at $t = 2$, the central bank is able to seize the banks' products.

However, we assume that the central bank cannot seize all of the banks' products. In fact, as in Kiyotaki and Moore (1997), it can seize only a fraction $\theta$ $(0 < \theta < 1)$ of bank products at $t = 2$. For this reason, the central bank provides only $\theta$ of fiat money at $t = 0$ to both banks. Thus, the banks issue $1 - \theta$ of banknotes at $t = 0$. In addition, as in the costly state verification model, if the central bank pays $\mathrm{C}$ to verify the state of both banks, it can seize all of the products produced by both of the banks.

In this situation, and contrary to the previous case, the merchants who want to consume the products of banks in a different region (i.e., $M_A^b$ and $M_B^a$) do not need to exchange their banknotes for each other. Instead, they demand the bank in the same region to transfer the banknote to the bank in the different region. That is, $M_A^b$, who holds the banknote of $B_A$, requires $B_A$ to transfer $D_A$ to $B_B$ in order to purchase the product of $B_B$. Therefore, the payment of $M_A^b$ and $M_B^a$ is settled through interbank settlement.

To compute the level of social welfare under the interbank settlement system, as in the previous section, we categorize this situation into four categories.

(1) Case 1: $\alpha = \beta = 1/3$

In this case, $\theta$ fraction of consumers in region A ($C_A$) pay their endowment good to $B_A$ and are repaid one unit of fiat money at $t = 0$. And $1 - \theta$ fraction of $C_A$ pay their endowment good to $B_A$ and are repaid $D_A$. At $t = 1$, a fraction $1 - \alpha$ of merchants in region A ($M_A^a$) find that they will want to consume the product produced by $B_A$. Although these merchants deliver their

[4] See the third and fourth terms in (1).

endowment good $m_A$ to $C_A$, θ of them receive one unit of fiat money from $C_A$ and $1-\theta$ receive $D_A$. On the other hand, a fraction α of merchants find that they will want to consume the product produced by $B_B$ at $t = 2$. As in the case of $M_A^a$, θ of $M_A^b$ receive one unit of fiat money and $1-\theta$ of $M_A^b$ receive one unit of $D_A$. The former can receive $b_B$ at $t = 2$ easily because these transactions have taken place with fiat money. But for the latter, to exchange banknotes for $b_B$, it is necessary that the interbank settlement system works well. That is, the merchants in region A (i.e., $M_A^b$) require $B_A$ to pay $D_A$ to $B_B$ and the merchants in region B (i.e.,$M_B^a$) require $B_B$ to pay $D_B$ to $B_A$. Using these transactions, merchants can receive the good produced by a bank in a different region. So, the total amount of $\alpha(1-\theta)D_A$ is transmitted to $B_B$. $B_B$ is also required to transfer the amount $\beta(1-\theta)D_B$ to $B_A$. Given $\alpha=\beta=1/3$, the amount transferred is equal. Therefore, $D_A = D_B = 1$ are satisfied and when these transactions are completed, all merchants receive the goods they want. In addition, as $\alpha=\beta$, the demands for $b_A$ and $b_B$ are 1 so that the markets for $b_A$ and $b_B$ again clear.

(2) Case 2: $\alpha=\beta=2/3$

In this, as in the previous case, the total amount of $\alpha(1-\theta)D_A$ is transmitted to $B_B$. $B_B$ is also required to transfer the amount $\beta(1-\theta)D_B$ to $B_A$. Given $\alpha=\beta=2/3$, the amount transferred to each other is equal, such these transactions are completed so that all merchants can receive the goods they want and the payments are settled without default. Therefore, $D_A = D_B = 1$ are satisfied. In addition, as $\alpha=\beta$, the demands for $\mathrm{b_A}$ and $\mathrm{b_B}$ are 1.

(3) Case 3: $\alpha=2/3$, $\beta=1/3$

Next, consider the case where $\alpha=2/3$ and $\beta=1/3$. As in the previous case, $M_A^b$ requires $B_A$ to pay $\alpha(1-\theta)D_A$ to $B_B$ and $M_B^a$ requires $B_B$ to pay $\beta(1-\theta)D_B$ to $B_A$. Given $\alpha>\beta$, $\alpha(1-\theta)D_A>\beta(1-\theta)D_B$ and $B_A$ does not have sufficient money to pay $B_B$ so that $B_A$ has to borrow $(1-\theta)(\alpha D_A-\beta D_B)$ from $B_B$ at $t = 1$. At $t = 2$, $B_A$ can acquire $(1-\alpha)\theta$ of fiat money from $M_A^a$ and $\beta\theta$ from $M_B^a$ so that the total fiat money $B_A$ receives at $t = 2$ is

$$(1-\alpha)\theta+\beta\theta=(1-\alpha+\beta)\theta=\frac{2}{3}\theta$$

If $(1-\theta)(\alpha \mathrm{D_A}-\beta \mathrm{D_B})>\frac{2}{3}\theta$ is satisfied, $B_A$ cannot repay $B_B$ by itself. In this situation, the central bank acts as the lender of last resort (LLR).[5] That is, $B_A$ has to borrow $\mathrm{R}=\frac{2}{3}\theta-(1-\theta)(\alpha \mathrm{D_A}-\beta \mathrm{D_B})$ from the central bank. If this lending by the central bank occurs, the

[5] If $(1-\theta)(\alpha \mathrm{D_A}-\beta \mathrm{D_B})<\frac{2}{3}\theta$ is satisfied, all transactions are settled through the interbank settlement system and the central bank does not have to act as the LLR.

interbank payment is settled and all merchants in both regions receive one unit of product produced by the banks. Therefore, their level of utility is $u(b_A) = u(b_B) = u(b)$. Thus, if the central bank acts as the LLR, merchants can avoid fluctuations in their level of consumption. However, it is not obvious whether the central bank acts as the LLR. Note that the central bank provides $\theta$ of fiat money at *t* = 0. Thus, if it plays the role of LLR, it should be repaid $\theta + R$ from $B_A$. The good produced by $B_A$ and $b_A$ is distributed $1 - \alpha$ to $M_A^a$ and $\beta$ to $M_B^a$ so that $1 - \alpha + \beta = 2/3$ units of $b_A$ are consumed and 1/3 of $b_A$ are not consumed. Given this, if the central bank pays C to seize all products, $1/3$ of $b_A$ can be seized. Thus, the central bank's payoff when it acts as the LLR is $1/3 - (\theta + R + C)$.

(4) Case 4: $\alpha = 1/3,\ \beta = 2/3$

This is the opposite case to the previous case. Thus, if the central bank acts as the LLR, all merchants receive the utility level $u(b_A) = u(b_B) = u(b)$ and the payoff of the central bank is $1/3 - (\theta + R + C)$.

From these four cases, the central bank chooses to act as LLR if the following inequality holds.[6]

$$2u(b) - \frac{1}{2}\left(\frac{1}{3} - (\theta + R + C)\right) \geq 2E_A[u_A]$$

The left-hand side is the social welfare when the central bank acts as the LLR. In this case, all merchants are able to avoid fluctuation in consumption but, as the second term on the LHS shows, the central bank bears some cost. The first part of this is the bankruptcy cost and the second part is the verification cost. This inequality is more likely to hold when $\theta$ and $C$ are both small and merchants are more risk averse.

## <u>The role of liquidity insurance</u>

In this section, we introduce a liquidity insurance market for banks and consider whether this market can improve the efficiency of the economy. To consider this problem, we can omit both cases 1 and 2 because neither bank faces a liquidity shortage in either case. Therefore, in the following discussion, we focus on case 3. In case 3, the central bank offers liquidity insurance as follows. If a bank pays an amount I as an insurance premium at *t* = 0 to the central bank, the central bank pays P as an insurance payout when the bank faces a liquidity shortage.

Then, from the discussion in Section 4, $B_A$ suffer a liquidity shortage of $\theta/3 + (1 - \theta)(\alpha D_A - \beta D_B)$. However, given liquidity insurance, $B_A$ will be paid P from the central bank at *t* = 1. Therefore, to cover this amount by liquidity insurance, the following equation should be

[6] We assume the central bank aims to maximize social welfare.

satisfied.

$$\{\theta/3 + (1-\theta)(\alpha D_A - \beta D_B)\} \times 1/2 = I \qquad (2)$$

Given this insurance, neither bank faces a liquidity shortage such that all transactions are settled. Therefore, $D_A = D_B = 1$ holds. Using these relations, we rewrite (2) as

$$\{\theta/3 + (1-\theta)/3\} \times 1/2 = I.$$

Thus, I = 1/6. Therefore, P = 2/6 = 1/3.[7] Consequently, under liquidity insurance, not only because all merchants in regions A and B consume one unit of the goods produced by the banks, but also because neither bank faces a liquidity shortage at $t = 1$, the central bank does not have to act as the LLR. In this sense, interbank settlement with liquidity insurance can both help avoid consumption fluctuations for agents and decrease the probability of a bank bailout. Therefore, this sort of financial system can improve the efficiency of the economy.[8]

## Conclusion

This paper considered how an interbank settlement system could improve social welfare. When only banknotes are able to circulate, the consumption by merchants may fluctuate so that welfare decreases. However, when both banknotes and fiat money are able to circulate, merchants can avoid these fluctuations in consumption, but only when the central bank acts as the LLR in which case the central bank should maintain the cost needed to sustain the interbank settlement system. Finally, when liquidity insurance is introduced, not only the consumption level of agents is smoothed, but the central bank can avoid the costs of sustaining the interbank settlement system.

To simplify the model, we assume that the central bank can seize some fraction, $\theta$, of bank products. Because of this assumption, we can describe the situation where both banknotes and fiat money coexist. In future research, we should examine the situation whereby the manner in which the central bank seizes these bank products is endogenously determined.

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[7] Note that the level of I is determined independently of the size of $\theta$.

[8] In this case, social welfare becomes 2u(b).

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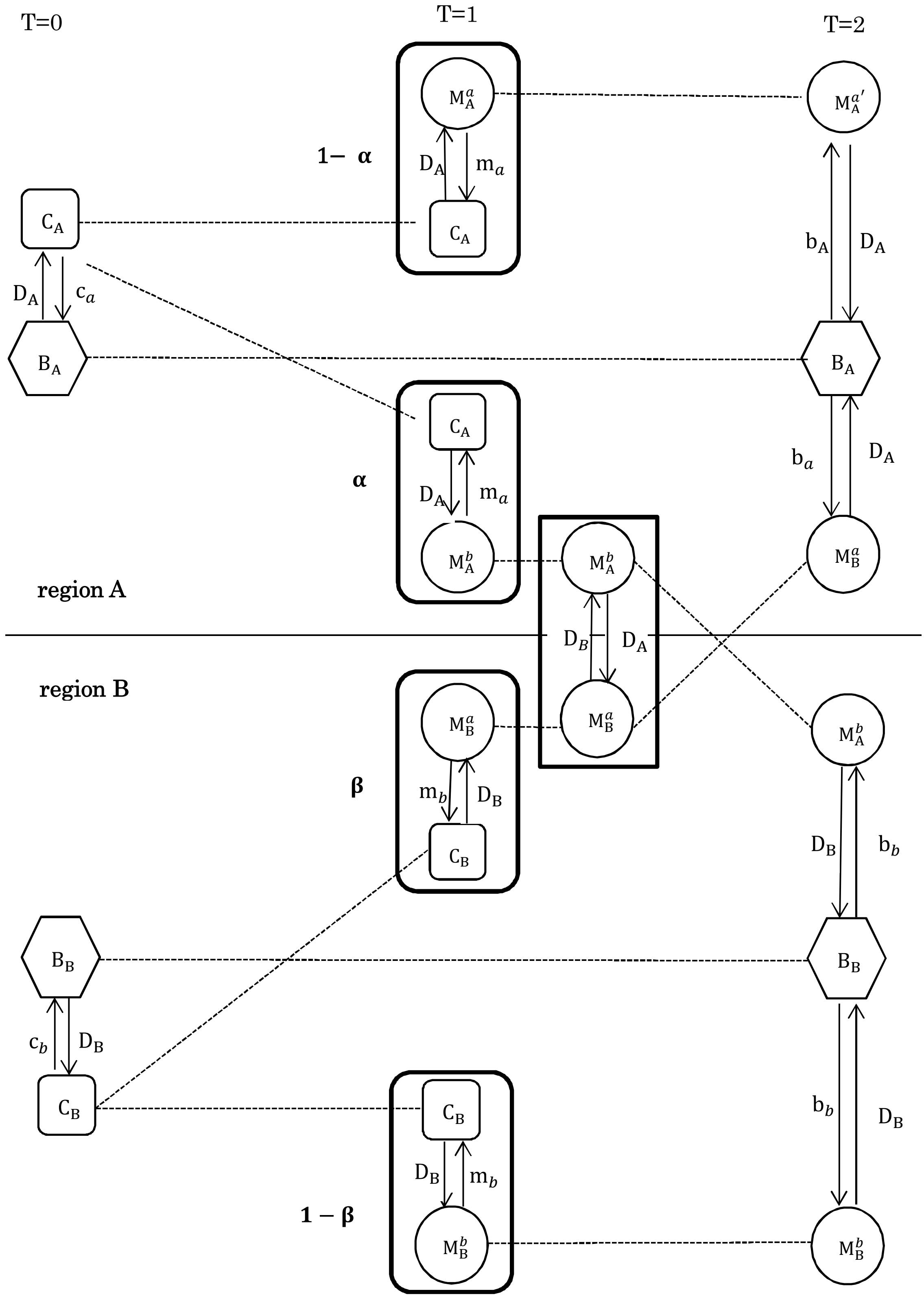

T=0
T=1
T=2
$1-\alpha$
$\alpha$
$\beta$
$1-\beta$
region A
region B
$M_A^a$
$M_A^{a'}$
$M_A^b$
$M_B^a$
$M_B^b$
$C_A$
$C_B$
$B_A$
$B_B$
$D_A$
$D_B$
$m_a$
$m_b$
$c_a$
$c_b$
$b_A$
$b_a$
$b_b$